Studentpad

JEE-MAIN 2021-22

Time : 90 Min	**Pre : Full Portion Paper**	**Marks : 120**

Hints and Solutions

01) Ans: **4)** $\frac{23}{\sqrt{17}}$

Sol: Since, the line L is passing through the point (13, 32).

Therefore, $\frac{13}{5}+\frac{32}{b}=1 \Rightarrow \frac{32}{b}=-\frac{8}{5} \Rightarrow b=-20$

The line K is parallel to the line L, then its equation must be $\frac{x}{5}-\frac{y}{20}=a$ or $\frac{x}{5a}-\frac{y}{20a}=1$

On comparing with $\frac{x}{c}+\frac{y}{3}=1$, we get

$20a=-3, c=5a=-\frac{3}{4}$

Hence, the distance between lines

$$=\frac{|a-1|}{\sqrt{\frac{1}{25}+\frac{1}{400}}}=\frac{\left|\frac{-3}{20}-1\right|}{\sqrt{\frac{17}{400}}}=\frac{23}{\sqrt{17}}$$

02) Ans: **1)** Wave moving in -x direction with speed $\sqrt{\frac{b}{a}}$

Sol: Given wave is $y(x,t)=e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$

$=e^{-\left(\sqrt{a}x+\sqrt{b}t\right)^2}$ It is a function of type

$y=f(\omega t+kx)=f\left(t-\frac{x}{v}\right) \therefore y(x,t)$ represent wave travelling along -x direction. Speed of wave

$=\frac{\omega}{k}=\frac{\sqrt{b}}{\sqrt{a}}=\sqrt{\frac{b}{a}}$.

03) Ans: **4)** H_2^{2-}

Sol: According to M.O.T. the viability of any molecule can be judged through the calculation of bond order.

Electronic	Configuration	Bond order
He_2^+	$\sigma_{1s}^2\sigma^*_{1s}{}^1$	$\frac{2-1}{2}=0.5$
H_2^-	$\sigma_{1s}^2\sigma^*_{1s}{}^1$	$\frac{2-1}{2}=0.5$
H_2^{2-}	$\sigma_{1s}^2\sigma^*_{1s}{}^2$	$\frac{2-2}{2}=0$
H_2^{2+}	σ_{1s}^2	$\frac{2-0}{2}=1$

The molecule having zero bond order will not be viable hence,

H_2^{2-} (option d) is the correct answer.

04) Ans: **3)** $\sqrt{2\,meV}$

Sol: As you can see in options, energy term is mentioned hence, we have to find out relation between $\frac{h}{\lambda}$ and energy. For this, we shall use de-Broglie wavelength and kinetic energy term in eV. de-Broglie wavelength for an electron $(\lambda)=\frac{h}{p}$

$\Rightarrow p=\frac{h}{\lambda}$ ...(i)

Kinetic energy of an electron=eV

As we know that, $KE=\frac{p^2}{2m}$

$\therefore eV=\frac{p^2}{2m}$ or $p=\sqrt{2meV}$ ...(ii)

From equation (i) and (ii), we get $\frac{h}{\lambda}=\sqrt{2meV}$

05) Ans: **2)** 2

Sol: Let

$I=\int_{\pi/4}^{3\pi/4}\frac{dx}{1+\cos x}=\int_{\pi/4}^{3\pi/4}\frac{1-\cos x}{1-\cos^2 x}dx$

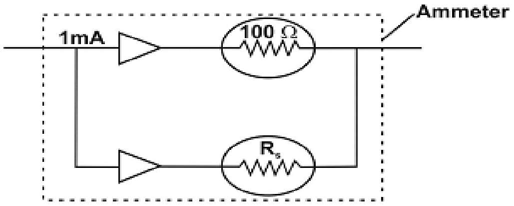
$=\int_{\pi/4}^{3\pi/4}\frac{1-\cos x}{\sin^2}dx$

$=\int_{\pi/4}^{3\pi/4}\left(\operatorname{cosec}^2 x-\operatorname{cosec} x \cot x\right)dx$

$=\left[-\cot x+\operatorname{cosec} x\right]_{\pi/4}^{3\pi/4}=\left[\left(1+\sqrt{2}\right)-\left(-1+\sqrt{2}\right)\right]=2$

06) Ans: **1)** $0.01\,\Omega$

Sol: Maximum voltage that can be applied across the galvanometer coil $=100\,\Omega\times10^{-3}$ A $=0.1$ V.



If R_s is the shunt resistance,

then $R_s\times10$ A $=0.1$ V $\Rightarrow R_s=0.01\,\Omega$

07) Ans: **3)** 2005 kHz 2000 kHz and 1995 kHz

Sol: Frequency associated with AM are

f_c-f_m, f, f_c+f_m

According to the question $f_c=2MHz=2000$ kHz

$f_m=5$ kHz

Thus, frequency of the resultant signal is/are carrier frequency $f_c = 2000\ kHz$, LSB frequency $f_c - f_m = 2000\ kHz - 5\ kHz = 1995\ kHz$ and USB frequency $f_c + f_m = 2005\ kHz$

08) Ans: **3)** $\beta \in (1, \infty)$

Sol: Let z=x+iy, given Re (z) =1

$\therefore x = 1 \Rightarrow z = 1 + iy$

Since, the complex roots are conjugate of each other.

$\therefore z = 1 + iy$ and $1 - iy$ are two roots of

$z^2 + \alpha z + \beta = 0$

Product of roots $= \beta \Rightarrow (1+iy)(1-iy) = \beta$

$\therefore \beta = 1 + y^2 \geq 1 \Rightarrow \beta \in (1, \infty)$

09) Ans: **2)** 12 A

Sol: Total power (P) consumed

$= (15 \times 40) + (5 \times 100) + (5 \times 80)$

$+ (1 \times 1000) = 2500\ W$

As we know, Power, $P = VI$

$\Rightarrow I = \frac{2500}{220} A = \frac{125}{11} = 11.3A$ Minimum capacity should be 12 A.

10) Ans: **4)** 95

Sol: Equation of tangent to the curve

$x^2 = 4ay$ at (x_1, y_1) is $xx_1 = 4a\left(\frac{y + y_1}{2}\right)$

Tangent to the curve $x^2 = y - 6$ at $(1, 7)$ is

$x = \frac{y+7}{2} - 6 \Rightarrow 2x - y + 5 = 0 \quad ...(i)$

Equation of circle is $x^2 + y^2 + 16x + 12y + c = 0$

Centre $(-8, -6) \quad \Rightarrow r = \sqrt{8^2 + 6^2 - c} = \sqrt{100 - c}$

Since, line $2x - y + 5 = 0$ also touches the circle.

$\therefore \sqrt{100 - c} = \left|\frac{2(-8) - (-6) + 5}{\sqrt{2^2 + 1^2}}\right|$

$\Rightarrow \sqrt{100 - c} = \left|\frac{-16 + 6 + 5}{\sqrt{5}}\right| \Rightarrow \sqrt{100 - c} = \left|-\sqrt{5}\right|$

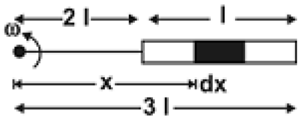
$\Rightarrow 100 - c = 5 \Rightarrow c = 95$

11) Ans: **1)** Cr

Sol: The substances which have lower reduction potentials are strong reducing agents. Therefore, Cr $\left(E^\circ_{Cr^{3+}/Cr} = -0.74\ V\right)$ is the strongest reducing agent among all the other given options.

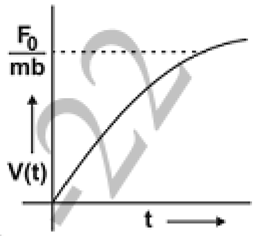
12) Ans: **1)** $\frac{5B\omega l^2}{2}$

Sol: Whenever a continuous shape is rotating in an uniform magnetic field, an emf is induced across it. To determine the induced emf, we take an elementary part, then emf induced in this elementary part is calculated. Now, for total induced emf in continuous shape is to be determined by integration within the limits. Consider small length of rod dx at a distance x from axial point of rotation.



The emf induced across the rod

$e = \int_{2l}^{3l} (\omega x) B dx = B\omega \frac{\left[(3l)^2 - (2l)^2\right]}{2} = \frac{5Bl^2\omega}{2}$



13) Ans: **3)**

Sol: As the force is exponentially decreasing, so its acceleration i.e., rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curve with decreasing slope with time. $a = \frac{dv}{dt} = \frac{F}{m} = \frac{F_0}{m} e^{-bt}$

$\Rightarrow \int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} dt$

$\Rightarrow v = \frac{F_0}{m}\left(\frac{1}{-b}\right) e^{-bt} \Big|_0^t = \frac{F_0}{mb} e^{-bt} \Big|_t^0$

$= \frac{F_0}{mb}\left(e^0 - e^{-bt}\right) = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$ with $V_{max} = \frac{F_0}{mb}$

14) Ans: **3)** 46.06 min

Sol: Half-life=6.93 min $\Rightarrow k_1 = \frac{0.693}{6.93} = 0.1$

We know, k_1 for per cent completion

$k_1 = \frac{2.303}{t} \log\left(\frac{a}{a - x}\right) \Rightarrow 0.1 = \frac{2.303}{t} \log\left(\frac{100}{100 - 99}\right)$

$0.1 = \frac{2.303}{t} \times \log \frac{100}{1}$

$0.1 = \frac{2.303}{t} \log 10^2 \quad \Rightarrow t = \frac{2.303 \times 2}{0.1} = 46.06$ m

15) Ans: **2)** $\frac{7}{2}$

Sol: Given planes are $2x + y + 2z - 8 = 0$

and $2x + y + 2z + \frac{5}{2} = 0$

Distance between two parallel planes

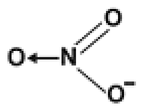
$= \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}} = \left|\frac{-8 - \frac{5}{2}}{\sqrt{2^2 + 1^2 + 2^2}}\right| = \frac{\frac{21}{2}}{3} = \frac{7}{2}$

16) Ans: **2)** sp^2, sp, sp^3

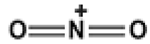
Sol: Count σ-bond, lone pairs and unpaired electron or count number of atoms directly

attached, lone pairs and unpaired electrons.
This is =2, then sp =5, then sp^3d
=3, then sp^2 =6, then sp^3d^2
=4, then sp^3

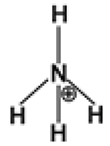
NO_3^- (1)



NO_2^+ (1l)



NH_4^+ (iii)



	σ-bond	Lone pair	Unpaired electron	Total
I.	3	x	x	$3(sp^2)$
II.	2	x	x	$2(sp)$
III.	4	x	x	$4(sp^3)$

Thus, correct answer is (b) sp^2, sp, sp^3 .
Alternate solution
Count the sum of outermost shell electrons of all the atoms and charge present at molecule. Now, divide the sum by eight and get the hybridisation. If sum is eight or less than eight then it is divided by two.

1. $NO_3^- \Rightarrow \frac{5+3\times6+1}{8} = \frac{5+18+1}{8} = \frac{24}{8} = 3(sp^2)$

1l. $NO_2^+ \Rightarrow \frac{5+2\times6-1}{8} = \frac{5+12-1}{8} = \frac{16}{8} = 2(sp)$

1ll. $NH_4^+ \Rightarrow \frac{5+4\times1-1}{2} = \frac{5+4-1}{2} = \frac{8}{2} = 4(sp^3)$

Thus, correct answer is (b) sp^2, sp, sp^3

17) Ans: **2)** $7\sqrt{2}$ units
Sol: Given, u=3i+4j; a=0.4i+0.3j From equation of motion, v=u+at $= 3\hat{i} + 4j + (0.4\hat{i} + 0.3\hat{j})10$

$\therefore$ Speed $= \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

18) Ans: **4)** 9
Sol: Stress Stress $= \frac{\text{Weight}}{\text{Area}}$

Volume will become (9^3)

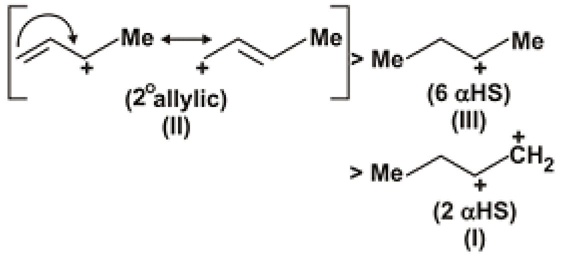
So weight=volume ×density ×g will also become $(9)^3$ times.

Area of cross section will become $(9)^2$ times.

$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$

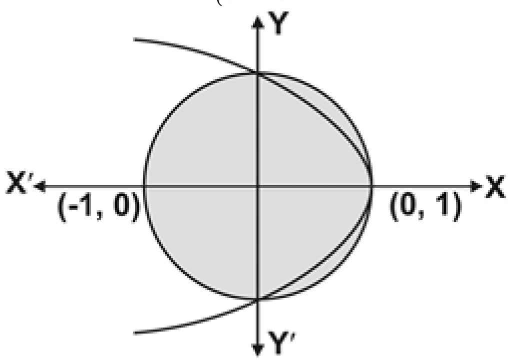
Hence, the stress increases by a factor of 9.

19) Ans: **1)** (II) > (III) > (I)
Sol: Higher the stability of carbocation, faster is the reaction because S_N1 reactions involve the formation of carbocation intermediate.



20) Ans: **4)** Statement I is true, Statement II is false.
Sol: Since, the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays
$KE_{photoelectron} = h\nu - \phi$
Stopping potential is to just stop the fastest photoelectron $V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$
So, KE_{max} and V_0 both increases. But KE ranges fro zero to KE_{max} because of loss of energy due to subsequent collisions before the electron to be ejected and not be due to range of frequencies of the incident light.

21) Ans: **1)** $\frac{\pi}{2} + \frac{4}{3}$
Sol: Given, $A = \{(x,y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$



Required area $= \frac{1}{2}\pi r^2 + 2\int_0^1 (1-y^2)dy$

$= \frac{1}{2}\pi(1)^2 + 2\left(y - \frac{y^3}{3}\right)_0^1 = \frac{\pi}{2} + \frac{4}{3}$

22) Ans: **4)** $\frac{dC}{dt} = K[A]$
Sol: This problem can be solved by determining the order of reaction w.r.t. each reactant and then writing rate law equation of the given equation accordingly as $r = \frac{dC}{dt} = k[A]^x[B]^x$
where, x=order of reaction w.r.t.A y=order of reaction w.r.t.B $1.2\times10^{-3} = k(0.1)^x(0.1)^y$
$1.2\times10^{-3} = k(0.1)^x(0.2)^y$ $2.4\times10^{-3} = k(0.2)^x(0.1)^y$

$R = k[A]^1[B]^0$

As shown above, rate of reaction remains constant as the concentration of reactant (B) changes from 0.1 M to 0.2 M and becomes double when concentration of A change form 0.1 to 0.2 (i.e., double).

23) Ans: **2)** 5

Sol:

$$R-NH_2 + CH_3-\overset{\overset{\large O}{||}}{C}-Cl \xrightarrow[(-HCl)]{} R-NH-\overset{\overset{\large O}{||}}{C}-CH_3$$

Since, each $-COCH_3$ group displaces one H atom in the reaction of one mole of $CH_3-\overset{\overset{\large O}{||}}{C}-Cl$ with one $-NH_2$ group, the molecular mass increases with 42 units. Since, the mass increases by (390-180)=210, hence the number of $-NH_2$ group is $\frac{210}{42} = 5$.

24) Ans: **4)** Na_3AlF_6 serves as the electrolyte

Sol: (a) In Hall-Heroult process for extraction of Al, carbon anode is oxidised to CO and CO_2.

(b) When Al_2O_3 is mixed with CaF_2, it lowers the melting point of the mixture and brings conductivity.

(c) Al^{3+} is reduced at cathode to form Al.

(d) Here, Al_2O_3 is an electrolyte, undergoing the redox process. Na_3AlF_6 although is an electrolyte but serves as a solvent, not electrolyte.

25) Ans: **4)** Statement I is true, Statement II is false

Sol: Given $\bar{x}$ is the AM and σ^2 is the variance of n observations $x_1, x_2, x_3, ..., x_n$.

AM of $2x_1, 2x_2, 2x_3, ..., 2x_n$.

$$= \frac{2x_1 + 2x_2 + 2x_3 + ... + 2x_n}{n}$$

$$= 2\left(\frac{x_1 + x_2 + x_3 + ... + x_n}{n}\right) = 2\bar{x}$$

Hence, it is obvious that Statement 2 is false. From here only we can tell that option (d) is the correct answer. However, here we will formally check the validity of Statement 1. Variance of

$2x_1, 2x_2, 2x_3, ..., 2x_n = \text{Variance }(2x_i)$

$= 2^2 \Rightarrow \text{Variance}(x_i) = 4\sigma^2$

Hence, Statement 1 is correct. Finally, Statement 1 is true and Statement 2 is false.

26) Ans: **2)** $-\frac{\pi}{2}$

Sol: Let $1 = \int_0^{\pi} [\cot x] dx$ ...(i)

$\Rightarrow 1 = \int_0^{\pi} [\cot(\pi - x)] dx = \int_0^{\pi} [-\cot x] dx$ ...(ii)

On adding Eqs.(i) and (ii),

$2l = \int_0^{\pi} [\cot x] dx + \int_0^{\pi} [-\cot x] dx = \int_0^{\pi} (-1) dx$

$[\because [x] + [-x] = -1, \text{ if } x \notin z \text{ and } 0, \text{ if } x \in z]$

$= [-x]_0^{\pi} = -\pi$

$\therefore 1 = -\frac{\pi}{2}$

27) Ans: **2)** 4.5%

Sol: $\therefore$ Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3}$ or $\rho = \frac{M}{L^3}$

$\Rightarrow$ Error in density $\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

So, maximum % error in measurement of p is

$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{3\Delta L}{L} \times 100$

or % error in density $= 1.5 + 3 \times 1$

% error $= 4.5\%$

28) Ans: **3)** 1×10^{-5}

Sol: $HQ = H^+ + Q^-$

$[H^+] = \sqrt{K_a C}$ by Ostwald's dilution law

$[H^+] = 10^{-pH} = 10^{-3} M$

$C = 0.1\ M$

Thus, $10^{-3} = \sqrt{K_a \times 0.1}$

$10^{-6} = K_a \times 0.1$

$\therefore K_a = 10^{-5}$

29) Ans: **3)** 2

Sol: We have, $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$

$$= \lim_{x \to 0} \frac{2\sin^2 x (3 + \cos x)}{x \times \frac{\tan 4x}{4x} \times 4x}$$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{(3 + \cos x)}{4} \times \frac{1}{\lim_{x \to 0} \frac{\tan 4x}{4x}}$$

$$= 2 \times \frac{4}{4} \times 1 \left(\begin{array}{l} \because \lim_{x \to 0} \frac{\sin\theta}{\theta} = 1 \\ \text{and } \lim_{x \to 0} \frac{\tan\theta}{\theta} = 1 \end{array}\right) = 2$$

30) Ans: **2)** 5

Sol: Given, $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\ \text{adj}\ A = AA^T$

Clearly, $A(\text{adj } A) = |A| I_2$

[$\because$ if A is square matrix of order n, then

$A(\text{adj } A) = (\text{adj } A).A = |A| I_n$]

$= \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} I_2 = (10a + 3b) I_2 = (10a + 3b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$ ...(i)

and $AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$= \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$ ...(ii)

$\because A(\text{adj } A) = AA^T$

$\therefore \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$

[using Eqs. (i) and (ii)]

$\Rightarrow 15a - 2b = 0 \Rightarrow a = \frac{2b}{15}$ ...(iii)

and $10a + 3b = 13$ ...(iv)

On substituting the value of 'a' from Eq.(iii) in Eq.(iv), we get $10.\left(\frac{2b}{15}\right) + 3b = 13$

$\Rightarrow \frac{20b + 45b}{15} = 13 \Rightarrow \frac{65b}{15} = 13 \Rightarrow b = 3$

Now, substituting the value of b in Eq.(iii), we get

$5a = 2$

Hence, $5a + b = 2 + 3 = 5$